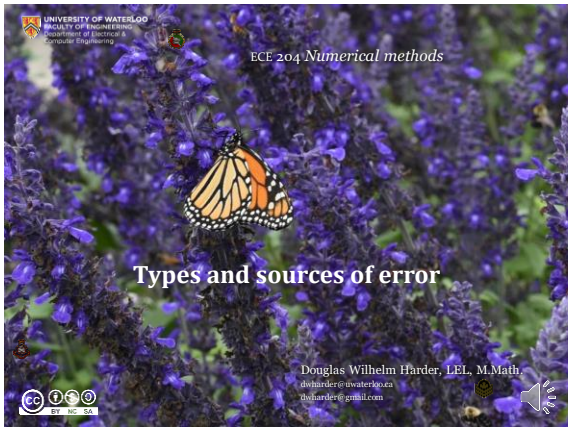

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Introduction

- In this topic, we will
  - Describe two types of error: random and systematic
  - Examine a number of sources of error and indicating whether their impact is random or systematic
  - Discuss how systematic errors must be accounted for in the design
  - Understand that in the next few lectures we will be looking at mitigating the effects of random error

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Types of error

- Measurements, including those from sensors and probes, will always be affected by errors
  - Measurements are a scalar value
    - A position vector or velocity vector is simply a pair or triplet of scalars, with each entry affected by the error
  - The effect of these errors allow them to be generally classified as one of two types:
    - Random errors
    - Systematic errors

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Random error

- A random error is a deviation to either side of the precise measurement
  - The errors will generally deviate around the true value
    - That is, the mean of such deviations has mean zero
  - It is correlated to the precision of the sensor or other device making the measurement
  - The more precise the sensor or other device, the smaller the magnitude of the deviation
    - Of course, the greater the precision, the higher the costs, including power and maintenance
  - The spread of such random errors can be described statistically
  - Another means of reducing the spread is to take multiple readings and average them

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Random error

- In your statistics course, you will learn about various statistical distributions to describe random phenomenon
  - Measurements tend to be affected by normally distributed ("bell curve") deviations around a mean value
  - Measurements of independent events relative to a periodic event tend to be uniformly distributed
    - If you randomly arrive at a subway stop without checking the schedule, and a train arrives 15 minutes, your wait will be uniformly distributed between 0 and 15 minutes
  - If your server receives approximately n independent customers per hour, the time between customers arriving will be $60/n$ minutes and the distribution will be a Poisson distribution

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Systematic errors

- A systematic error is one that is deviation that is consistent between measurements
  - Their consistency makes them difficult to detect statistically
  - They are however, reproducible
    - They can be accounted for in the design
    - For example, subtracting a corrective factor from measurements or taking steps to eliminate the error from the sensor or other device
  - This is often described as a bias

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Sources of error

- We will now look at sources of error
  - We will describe how the source of the error may be systematic or random

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Floating-point arithmetic

- The least-significant bits of floating-point numbers tend to be randomly distributed
  - Lack of random distributions in such digits is one source of detecting fraud
- Consequently, many floating-point calculations result in the introduction of random errors
  - Serious consequences usually only occur when:
    - Similar numbers are being subtracted
    - Significantly different numbers are being added

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Floating-point arithmetic

- Recall that for rounding,
  if the digits beyond the last were exactly 50000…
  or the bits beyond the last were exactly 10000…
  we selectively rounded up or down based on the parity of the least significant digit
  - If this was not done, this would result in a systematic error
- With addition of decimal digits, this is less common
  - Consider, however adding two numbers $1 \leq m, n < 2$ in binary
    - Each has the same exponent
    - The sum must be greater than or equal to 2

$$\begin{array}{r} 1.0001010\cdots00111110? \\ +\ \underline{1.0110101\cdots10011101?} \\ 10.0111111\cdots11011011? \end{array}$$

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Model error

- Model errors are issues with your modeling of the actual system
  - Modeling errors tend to be systematic
  - When you test or deploy your solutions,
    modelling errors will likely reveal themselves
  - Modeling errors may, however, be random
    - Failing to take into account vibrations and shielding sensors from such vibrations

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Environmental error

- Nature is not constant
  - Local turbulence, heat and other chaotic factors will cause random errors in readings
    - Such errors can never be eliminated and must be accounted for
  - Environmental factors that are systematic are a failure in the model

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Drift errors

- In some cases, a measurement may drift
  - Clocks that start synchronized will, over time, drift apart, as you are likely not using atomic clocks
    - Such drifts will result in systematic errors
  - Algorithms exist to allow numerous devices to synchronize their clocks periodically even if there are failures in one of the clocks or an error in the communication of the current time
  - Similarly, if you are recording angular velocity and integrating this to get the change in angle, over time, you will have a bias in your direction

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Production errors

- Any sensor or other device will not be perfect
  - Assuming there is quality control, there will never-the-less be a random error in each sensor or other device that is produced
  - Such errors can be reduced with either:
    - Higher costs by purchasing more consistent devices
    - Appropriate algorithms to mitigate the additional randomness
- For example, an inexpensive low-pass filter requires a resistor and a capacitor
  - The circuit elements have specified resistances and capacitances, but there is always an error
  - A design that requires very precise elements will cost more

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Calibration errors

- Calibration is the process of comparing a device to a standard
  - Failure to correctly adjust a device after calibration may result in a systematic error
  - Deviations from the standard may be:
    - Considered to be acceptable or within limits
    - Corrected, if possible
    - Compensated for in the algorithms

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Lag-time errors

- Some sensors take time to adjust before a reading can be made
  - Reading such sensors before results in a locally systematic error
    - Consider for example, a thermometer reading a temperature
      - If the ambient temperature where the thermometer is stored is above that of what is being measured, the reading will be too high
      - The converse will result in readings that are too low
  - Devices with inductors, or other magnetic devices, also take time to adjust to new conditions
  - A 10-day average of individuals with an illness will take a few days to react to a significant up-swing or down-turn in infections

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The effect of types of error

- Returning to our classifications:
  - Random errors tend to affect the precision of our measurements
  - Systematic errors tend to affect the accuracy
- To deal with errors:
  - Your designs should account for systematic error
  - Our algorithms will mitigate the effects of random error

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Consequence

- One could devote a single high-precision device to continually take readings of one measurement, but this will become prohibitively expensive
  - The goal of engineering is to respond economically
  - Instead, sensors readings and measurements are made periodically
  - The period is chosen to ensure that no hard real-time limits are breached
  - Periodic sampling is the only approach that can be analyzed mathematically
    - You will see more on this in your course on linear systems and signals
  - This analysis will allow you to use less precise devices

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Consequences

- Thus, regardless of what sensor is being read, your periodic data will be less than perfectly precise, even if you account for all systematic error

  - In devising numerical algorithms, we must also take these errors into account
  - We will also look at how to deal with such errors
    - More on this will be covered in your statistics and linear systems courses, specifically the *z-transform*

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One solution

- If your readings, or discrete-time signal, consists of an approximately constant signal together with only normally-distributed errors with mean zero,
  the single most efficient means of eliminating the error or noise introduced is to take a running average of the last *n* samples
  - There is no algorithm more suited to eliminating such noise
  - Problem: this does not work if there is the signal is not constant

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Summary

- Following this topic, you now
  - Understand the two types of error: random and systematic
  - Have considered various sources of error and described these sources as having systematic or random error
  - Are aware that systematic errors must be taken account of during the design phase
  - Understand that we will be looking at mitigating the effects of random errors, including but not limited to random errors introduced by floating-point arithmetic

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References

[1] https://en.wikipedia.org/wiki/Observational_error

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Acknowledgments

None so far.

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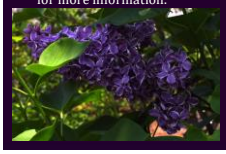
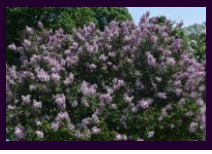
Colophon

These slides were prepared using the Cambria typeface. Mathematical equations use Times New Roman, and source code is presented using `Consolas`. Mathematical equations are prepared in MathType by Design Science, Inc. Examples may be formulated and checked using Maple by Maplesoft, Inc.

The photographs of flowers and a monarch butter appearing on the title slide and accenting the top of each other slide were taken at the Royal Botanical Gardens in October of 2017 by Douglas Wilhelm Harder. Please see
https://www.rbg.ca/
for more information.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